**UNIVERSITY OF MANITOBA**
**Term Test 3B**
COURSE: MATH 1500
DATE & TIME: November 26, 2018, 5:40PM – 6:40PM
CRN: various
DURATION: 1 hour EXAMINER: various

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# INSTRUCTIONS

I. No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

II. This exam has a title page, 4 pages of questions and two blank pages for rough work. Please check that you have all the pages. **DO NOT REMOVE THE SCRAP PAPER**

III. The value of each question is indicated in the lefthand margin beside the statement of the question. The total value of all questions is 50 points.

IV. **Answer all questions on the exam paper** in the space provided beneath the question. Unjustified answers will receive little or no credit. Only techniques taught in this course should be used. **Do not continue on the back of the page.** If you need more space, continue on one of the scrap pages, CLEARLY INDICATING THAT YOUR WORK IS TO BE CONTINUED.

V. Do not deface the QR - code in the top right corner. Doing so may result in the page not being scanned and therefore not graded.

| Question | Points | Score |
|---|---|---|
| 1 | 4 | |
| 2 | 2 | |
| 3 | 7 | |
| 4 | 6 | |
| 5 | 9 | |
| 6 | 22 | |
| Total: | 50 | |

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[4] 1. Let $h(x) = (2g(x) - x)(2f(x) - 2)$, Find $h'(3)$ if $f'(3) = g'(3) = 4$ and $f(3) = g(3) = -4$.

**Solution:**
Using the product rule for derivatives,

$$\begin{aligned} h'(x) &= (2g(x) - x)'(2f(x) - 2) + (2g(x) - x)(2f(x) - 2)' \\ &= (2g'(x) - 1)(2f(x) - 2) + (2g(x) - x)(2f'(x)) \end{aligned}$$

Therefore,

$$\begin{aligned} h'(3) &= (2g'(3) - 1)(2f(3) - 2) + (2g(3) - 3)(2f'(3)) \\ &= (2(4) - 1)(2(-4) - 2) + (2(-4) - 3)(2(4)) = -154 \end{aligned}$$

[2] 2. Given that, for a function $f(x)$, $f''(x)$ is continuous near $x = c$, $f'(c) = 0$, and $f''(c) < 0$, what can we conclude about $f(c)$ in terms of relative (local) extrema? Explain your answer.

**Solution:** Since $f'(c) = 0$, $f$ is defined at $x = c$ and has a critical number at $x = c$. Since $f''(c) < 0$, according to the second derivative test, $f$ has a relative (local) maximum at $x = c$.

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[7] 3. Find the absolute extrema of $f(x) = x^3 - 12x + 1$ on the interval $[-3, 5]$. Justify your answer.

**Solution:** Since $f$ is a polynomial function, it is continuous on the closed interval $[-3, 5]$.

$$f'(x) = 3x^2 - 12$$

which is defined everywhere.
To find critical numbers, we have to solve $f'(x) = 0$.

$$f'(x) = 0 \Rightarrow 3x^2 - 12 = 0 \Rightarrow 3(x^2 - 4) = 0 \Rightarrow x = -2 \text{ or } x = 2.$$

Hence, with the endpoints, we test $x = -2$ and $x = 2$, since they are both in the interval.

$$\begin{aligned} f(-2) &= 17 \\ f(2) &= -15 \\ f(-3) &= 10 \\ f(5) &= 66. \end{aligned}$$

Then, the absolute maximum is 66 and the absolute minimum is $-15$.

[6] 4. Suppose that $f(x)$ is differentiable on an interval $I$ and $f'(x) > 0$. Show that $f(x)$ is increasing on $I$.

**Solution:** Let $x_1$ and $x_2$ be any two distincts numbers in $I$ such that $x_2 > x_1$. Since $f'(x)$ exists on $I$, $f(x)$ is differentiable on $I$ and thus it is continuous on $I$. Therefore, we have that $f(x)$ is continuous on $[x_1, x_2]$ and differentiable on $(x_1, x_2)$.
So by the Mean Value Theorem, there exists $c$ between $x_1$ and $x_2$ such that

$$f(x_2) - f(x_1) = f'(c)(x_2 - x_1).$$

Since $f'(c) > 0$ and $x_2 - x_1 > 0$, we conclude that $f(x_2) - f(x_1) > 0$ and, hence, $f(x_2) > f(x_1)$. Thus, $f(x)$ is increasing on $I$.

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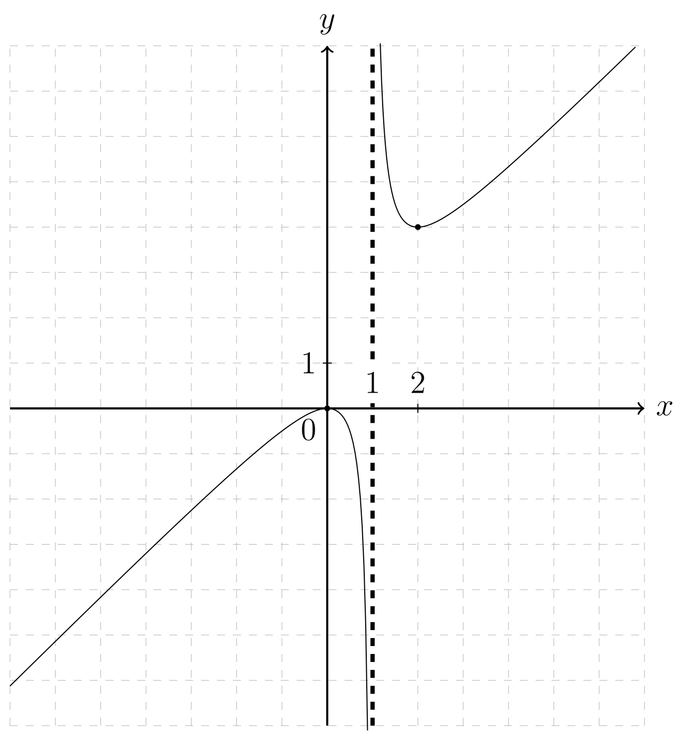
5. The graph of the function $f(x) = \dfrac{x^2}{x-1}$ is given below. Use this graph to gather information and fill in the blanks. (*If a feature doesn't apply, write "None."*)




[1] (a) Equation(s) of any vertical asymptotes: $\underline{x = 1}$

[1] (b) Equation(s) of any horizontal asymptotes: None

[1] (c) Open intervals where $f$ is increasing: $(-\infty, 0)$, $(2, \infty)$

[1] (d) Open intervals where $f$ is decreasing: $(0, 1)$, $(1, 2)$

[1] (e) $x$ and $y$-coordinates of any local maxima: $(0, 0)$

[1] (f) $x$ and $y$-coordinates of any local minima: $(2, 4)$

[1] (g) Open intervals where $f$ is concave up: $(1, \infty)$

[1] (h) Open intervals where $f$ is concave down: $(-\infty, 1)$

[1] (i) $x$ and $y$-coordinates of any inflection point(s): None

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6. Use the function $f(x)$, the first derivative $f'(x)$ and the second derivative $f''(x)$ as defined here to gather information and fill in the blanks below. (*If a feature doesn't apply, write "None."*)

$$f(x) = \frac{x^2+1}{x^2-9} \qquad f'(x) = \frac{-20x}{(x^2-9)^2} \qquad f''(x) = \frac{60(x^2+3)}{(x^2-9)^3}$$

[1] (a) Domain of $f$: $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

[1] (b) Symmetry of $f$: Even

[1] (c) $x$-intercepts: None

[1] (d) $y$-intercept: $-1/9$

[1] (e) Equation(s) of any vertical asymptotes: $x = -3, x = 3$

[1] (f) Equation(s) of any horizontal asymptotes: $y = 1$

[1] (g) $x$ and $y$-coordinates of any critical point(s): $(0, -1/9)$

[2] (h) Open intervals where $f$ is increasing: $(-\infty, -3)$, $(-3, 0)$

[2] (i) Open intervals where $f$ is decreasing: $(0, 3)$, $(3, \infty)$

[1] (j) $x$ and $y$-coordinates of any local maxima: $(0, -1/9)$

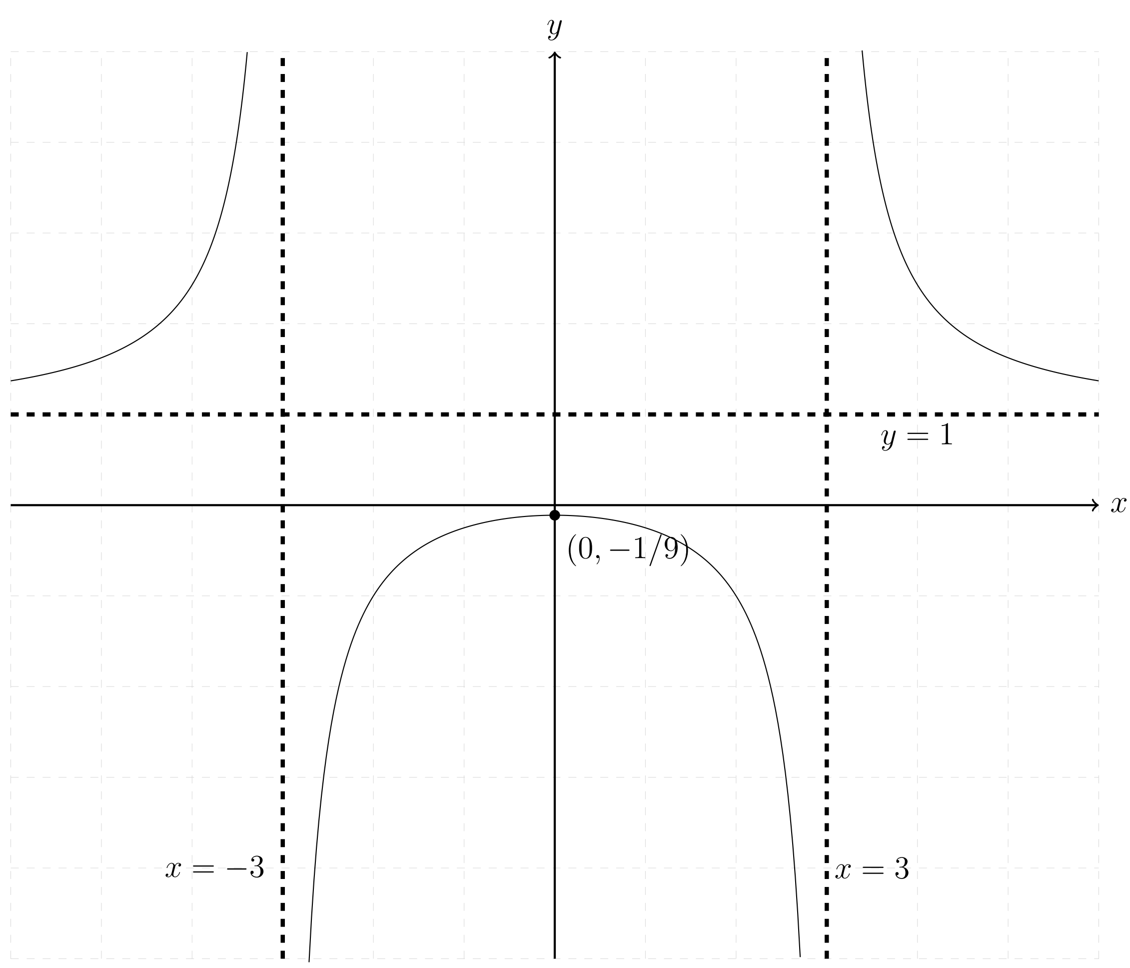
[1] (k) $x$ and $y$-coordinates of any local minima: None

[2] (l) Open intervals where $f$ is concave up: $(-\infty, -3)$, $(3, \infty)$

[2] (m) Open intervals where $f$ is concave down: $(-3, 3)$

[1] (n) $x$ and $y$-coordinates of any inflection point(s): None

[4] (o) Use the information from the previous parts to give a neat sketch of the graph $y = f(x)$, making sure that you label all important features of the graph.



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**If you are using this page to continue your work from a previous question, clearly indicate on the original page that your work is continuing here. Otherwise, your work will not be marked.**

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